

## Capability of f-k and SPAC methods in determining shear wave velocity of subsurface soil

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### Abstract

A series of microtremor array measurement was performed in a site of Tehran (shaghayeg park) with the aim of estimating the shear wave velocity ( $V_s$ ) profile for near surface layers. The SPAC and f-k array processing technique were used and the results were compared with other information specially a 200 meters depth borehole and the down-hole and PS logging data have been done in this paper. The data of different time, day and night were processed and the results were compared with the actual  $V_s$  profile by definition the various criteria. We found that, the SPAC method is probably more convenient compared to the f-k method. SPAC method gives results as good as the f-k method while using smaller number of recording stations and shorter array dimensions.

**Key words:** Shear wave velocity, microtremor, f-k method, SPAC method.

### 1. Introduction

Accurate evaluation of the S-wave velocity profile of the sub-layers (down to seismic bedrock) has particular importance on site effect studies and other domain of geotechnical earthquake engineering. Most of the cities and urban areas of the world are founded on soft sediments (valleys, recent deposits, estuaries ...) the soil structure of which is prone to amplify seismic waves (Bard 1994, Murphy et al. 1988). One of the most valuable geological properties of the soil is shear-wave velocity. Today, by using nondestructive methods, we can determine the shear-wave velocity of the subsurface layers of the earth. Using microtremor measurement (single station and array) to measure shear wave velocity of

subsurface layers of the earth have attracted the attention of many researchers and engineers in the recent years, due to their lower cost and lesser time consuming, compared to other  $V_s$  investigation methods such as boring and direct measurements. The single station microtremor technique is based on using the H/V technique, introduced by Nakamura (Nakamura 1989). This technique gives almost precisely the natural frequency of sedimentary deposits. However, it has some limitations in estimating the  $V_s$  profile due to its simultaneous dependency on sediment thickness and shear wave velocity. The microtremor array methods based on dispersive properties of surface wave are therefore introduced as the alternative technique, which could

give the  $V_s$  profile of a site. There are two general categories of array processing techniques, the frequency-wavenumber (f-k) and the spatial autocorrelation (SPAC) methods. In this paper using some direct geotechnical/geophysical tests, the real  $V_s$  profile of the studied site has been determined. Afterward  $V_s$  profile calculated using f-k and SPAC methods. Finally the results compared with real  $V_s$  profile in order to investigate capability of the microtremor array method in estimating shear wave structure of subsurface soil up to 200 meters depth.

**2. Data Acquisition and Processing**

Figure 1 shows the location of studied site and the arrangement of stations in different arrays processed during this study. The site was located in south of Tehran, in Shaghayegh park. The geography of Tehran is a large plain in the southern central area of the Alborz Mts. This plain consists of northerly-southerly slopes and divides into different parts by easterly-westerly high and low lands. The parts include varying altitudes and marginal folds of the Alborz Mts., the mountain slopes of Tehran and the plain of Tehran, respectively, from north to south. Sediments of the Tehran ground are relatively recent alluvium formations that came down along rivers

and flood plains after the origins of the Alborz Mts. (Berberian et al. 1985).

Rieben (Rieben 1966) divided those sediments into formations and titled as A, B, C and D, whereon A represents the oldest and grading through to D which represents the most recent formed alluvium. This paper focuses on the D sediments of southwest Tehran. These youngest sediments are the results of deposits from rivers, run off channels, alluvial fans and young cones and are consisted of mainly fine-grained (clay and silt) sediments. Fifteen CMG-6TD Guralp seismometers were used simultaneously as 3 concentric circles arrays with a radius of 25, 35 and 50m, named as array A, B, C respectively. An additional array E which consisted of two concentric triangles is also considered that shown on the Figure 1. The arrays were installed for 15 hours continuously, from 16:30 hr on March 28, 2007 to 7:30 hr on the following day. A sampling rate of 100 sample/sec was used for all seismometers. SPAC method was used for processing the data recorded in arrays A, B and C. For array E the F-K method was utilized. SESARRAY software package (<http://www.geopsy.org>, Wathelet et al. 2005) was used to perform data processing.

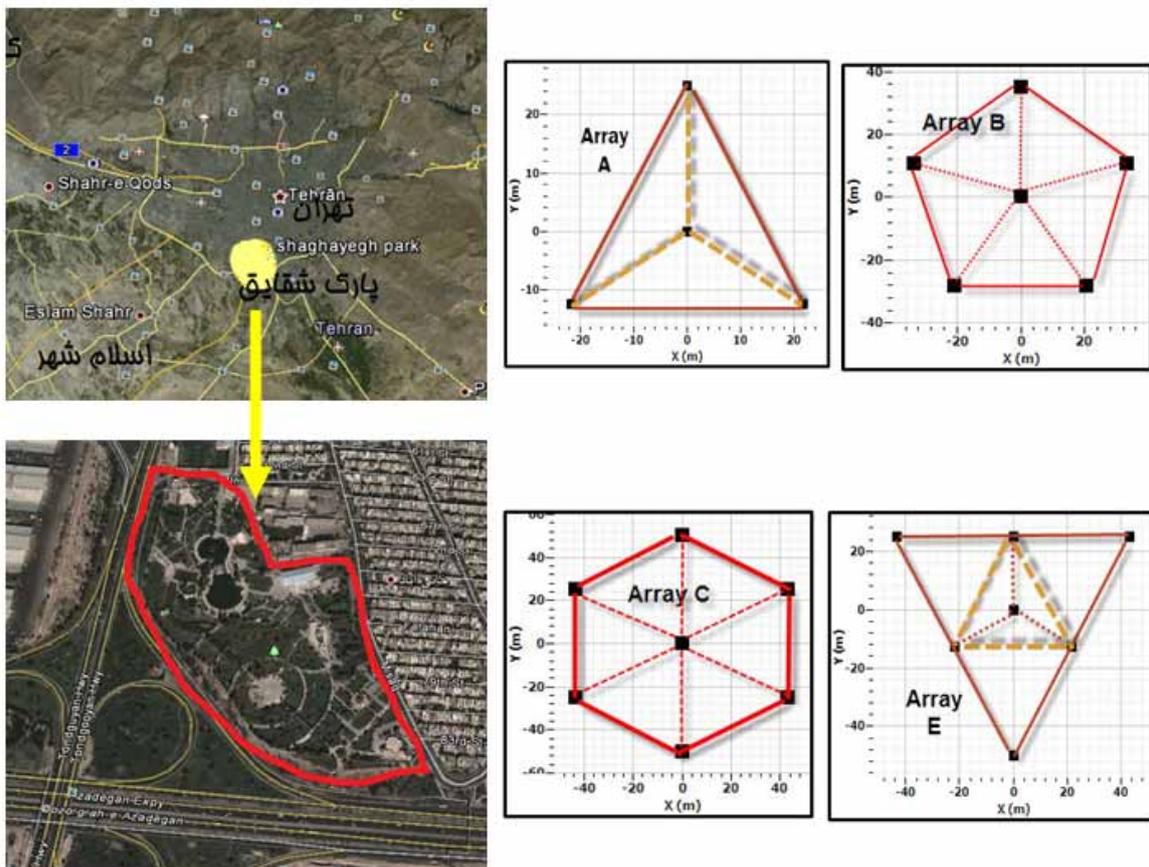


Fig. 1 Location of studied site on Tehran map (left plan) and the used array geometry (right plan).

Aki (Aki 1957, 1965) developed a technique to deduce a phase-velocity dispersion curve from the microtremors recorded by a seismic array. He established that the spatial cross-correlation coefficient as a function of frequency for a given interstation distance,  $r$ , and angular frequency,  $\omega$ ,  $\rho(r, \omega)$ , averaged over many different azimuths,  $\tau$ , can be written as

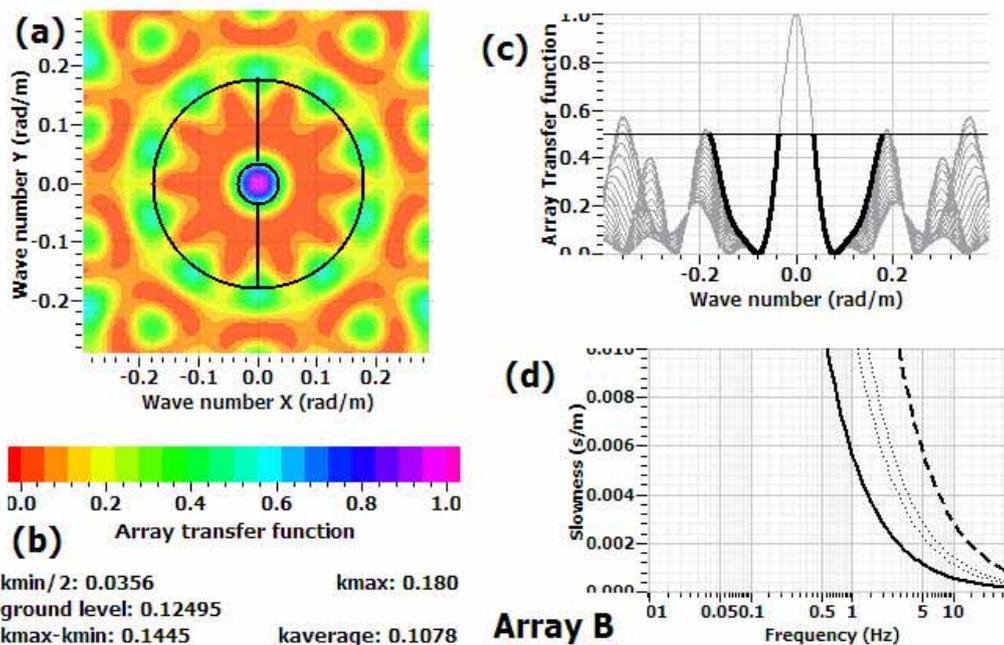
$$\rho(r, \omega) = \frac{1}{2\pi\phi(r=0, \omega)} \int_0^{2\pi} \phi(r, \theta, \omega) d\theta = J_0\left(\frac{r\omega}{c}\right), \quad (1)$$

where  $\rho(r=0, \omega)$  is the average autocorrelation function at the center of the array,  $\phi(r, \theta, \omega)$  is the cross-correlation function between the record at a site at coordinates  $(r, \theta)$ , and the record obtained at the station at the origin,  $c$ , is the phase velocity at frequency  $\omega$  at the site, and  $J_0$  is the Bessel function of first kind and order zero. The wave field is assumed to consist of surface waves propagating with equal power in all directions. The only unknown in the preceding equation is the phase velocity for each frequency, which can be obtained from the inversion of the observed correlation coefficients. In turn, it is possible to invert that phase velocity dispersion curve to obtain a shear-wave velocity profile with standard techniques (e.g.,

Herrmann 1987). The details of the method have been published several times (e.g., Asten 1976, Chouet 1998). Frequency–wavenumber (f-k) method (Capon 1969, Lacoss et al. 1969) is based on the fact that a stationary random process can be characterized by means of a spectral density function, which provides the information concerning the power as a function of frequency. In a similar manner, seismic noise can be characterized by a frequency–wavenumber spectral density function, which provides the information concerning the power as a function of frequency and the vector velocities of the propagating waves. The goal is to derive the different wave velocities and directions of approach as a function of frequency from the frequency–wavenumber spectral density function of the micro-tremor. For that, the stationary assumption in both time and two spatial coordinates has to be accomplished. Theoretical array response could be used as other criteria for determining the reliable part of dispersion curve. The theoretical response of array B is shown in Figure 2 and the calculated values of  $k_{min}$  and  $k_{max}$  for different array are presented in Table 1.

**Table 1.  $k_{max}$  and  $k_{min}$  values for used arrays in the Tehran site, Shaghayegh park.**

Array Name	$k_{min}/2$	$k_{max}$	Ground Level	$k_{max}-k_{min}$	Kaverage
A	0.0531	0.2373	0.1969	0.184284	0.145242
B	0.0356	0.1801	0.1249	0.144472	0.107802
C	0.0289	0.1178	0.0919	0.088928	0.073372
E	0.0317	0.1510	0.1239	0.119333	0.0913735



**Fig. 2 (a) Theoretical array response in  $(k_x, k_y)$  plane for 6 sensors (array B) in Tehran site. (b) Value of array transfer function and  $k$ . (c) The sections through (a) response for various propagation directions that shown by grey curves. (d)  $k_{max}$  and  $k_{min}$  values in slowness-frequency domain.**

As an example results of SPAC method in studied site, the rings of pair stations and the SPAC coefficients curve for array C are presented in

Figures 3 and 4, respectively. The frequency range was selected as 1.5-2.5 Hz.

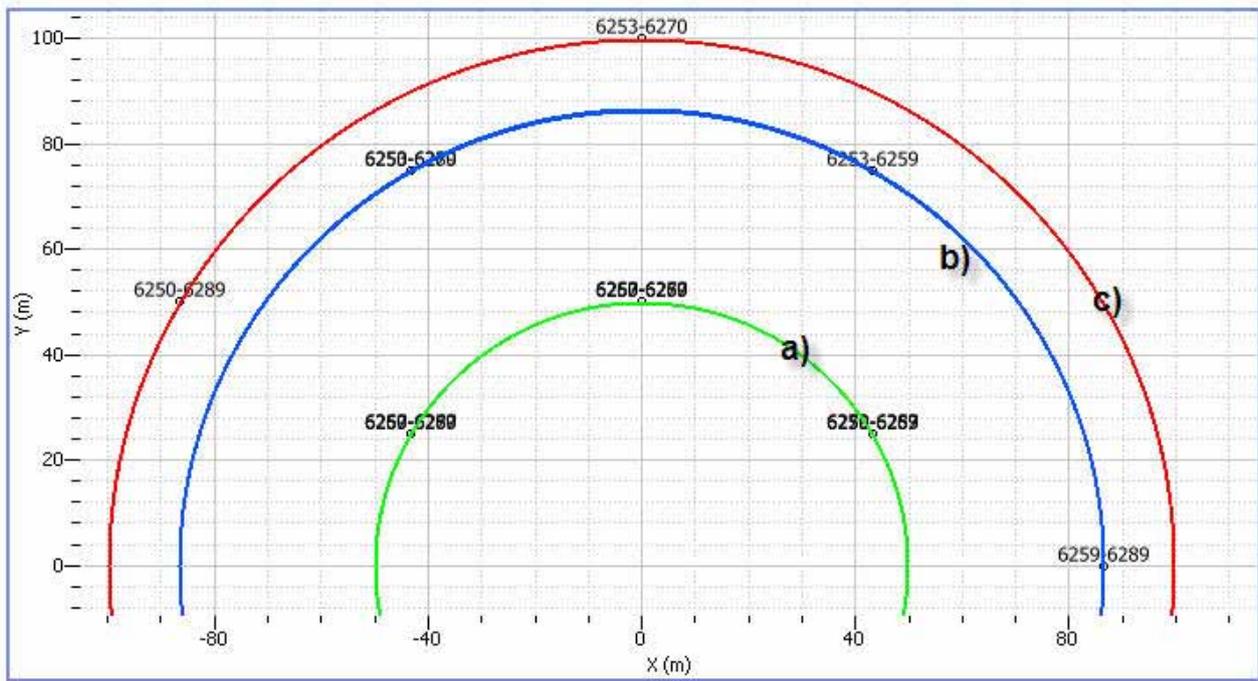


Fig. 3 Selected rings and possible combinations of pair stations azimuth-distance in array C: a) green: 50m, b) blue: 86.6m, c) red: 100m.

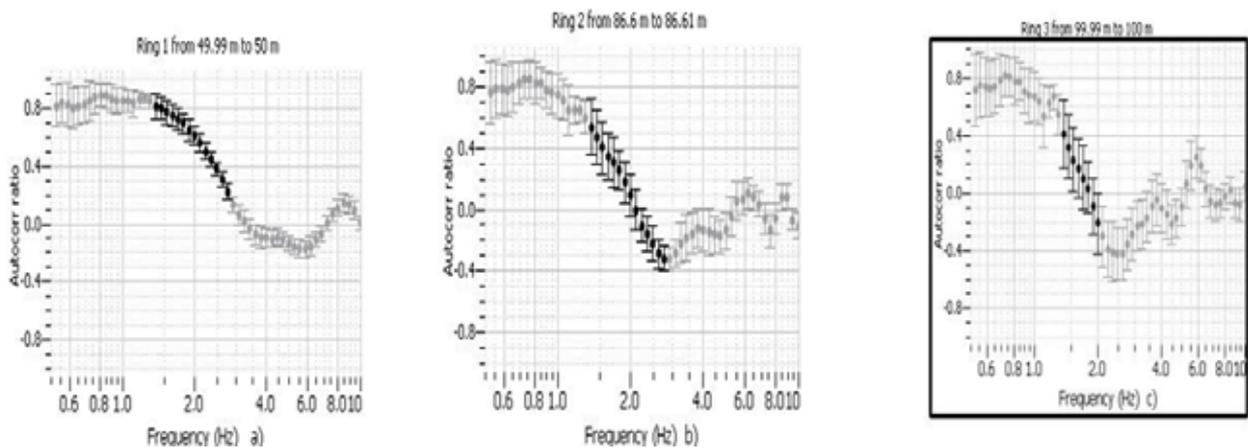


Fig. 4 SPAC curves for array C in 0.95-18Hz frequency range and rings of Fig. 3 a) ring 50m, b) ring 86.6m, c) ring 100m.

### 3. Inversion results

To obtain the shear wave profile of subsurface layers inversion of dispersion curve was performed using the neighborhood algorithm (Wathelet et al. 2004) supported by SESARRAY package. Different

input models were considered for a parametric study. One of the suggested models is presented in Table 2(a). The measured average  $V_s$  profile of the site, derived from down-hole seismic measurement and PS-logging are also presented in Table 2(b).

Table 2. a) One of suggested models for underground layers based on different limits of Vs profile to perform inversion operation b) the measured Vs profile of the site base on direct methods (Tehran site, Shaghayegh park).

a)			b)	
Number of layers	Depth of layers( range) (m)	Vs(range) (m/s)	Depth of layers (m)	Vs (m/s)
6	1-12	150-400	0-5	120
	2-30	200-500	5-13	260
	3-60	300-600	13-28	360
	4-100	400-700	28-60	400
	5-160	450-700	60-160	600
	>160	600-1000	>160	700

An example of inversion results is shown in Figure 5 (for array C in 21-22 (GMT+00)). Variation of Vs against depth and slowness against frequency (for fundamental mode) presented in (5-a) and (5-b),

respectively. Also variation of Vs range in six layers from proposed model is shown in Figure 6 against misfit values (space parametric).

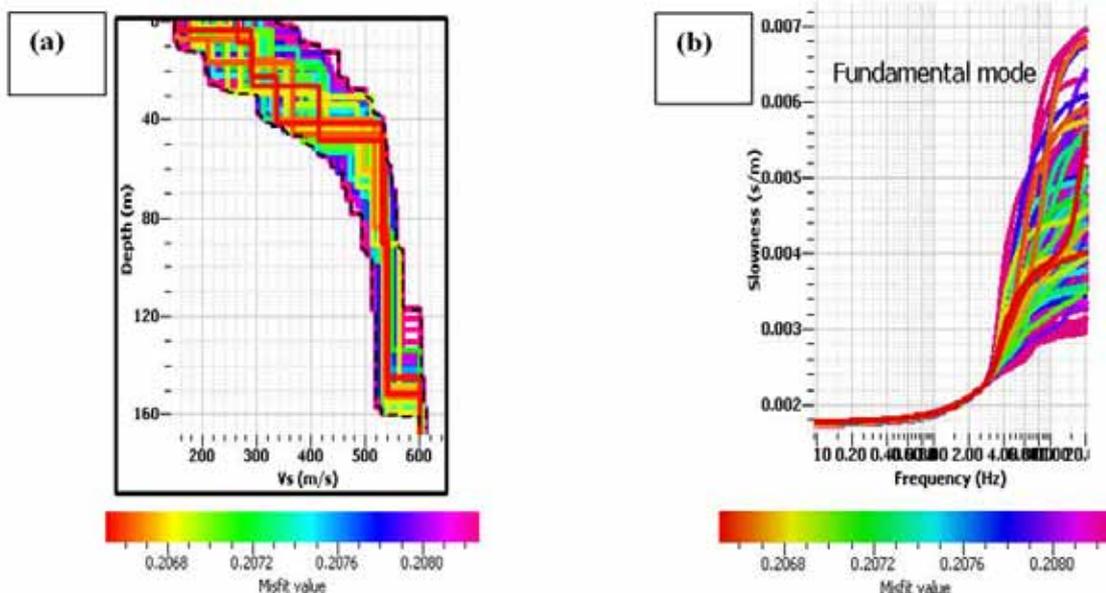


Fig. 5 Array C, the results of inversion processing: a) Derived Vs for models b) Derived dispersion curve

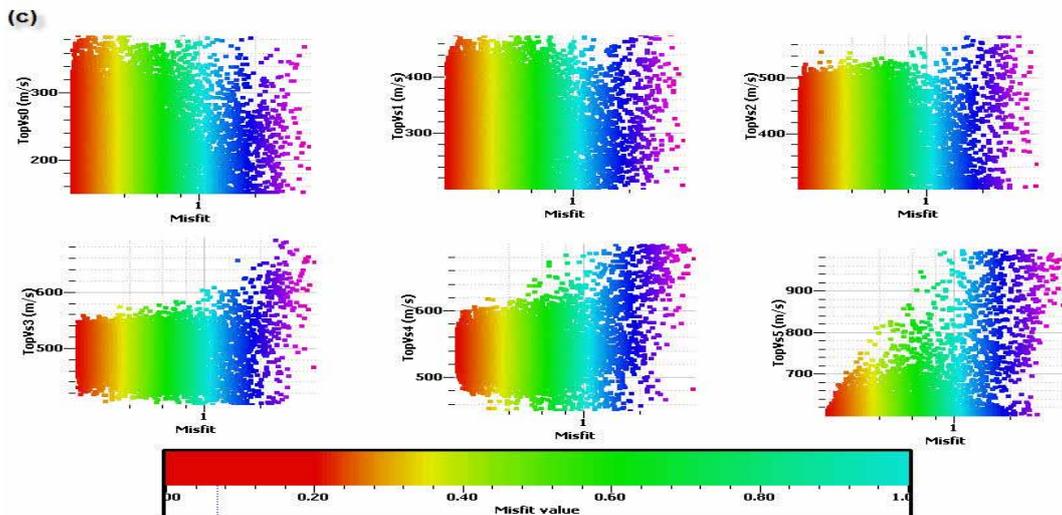


Fig. 6 Array C, the results of inversion processing: Variation of Vs range in six layers from proposed model against misfit values

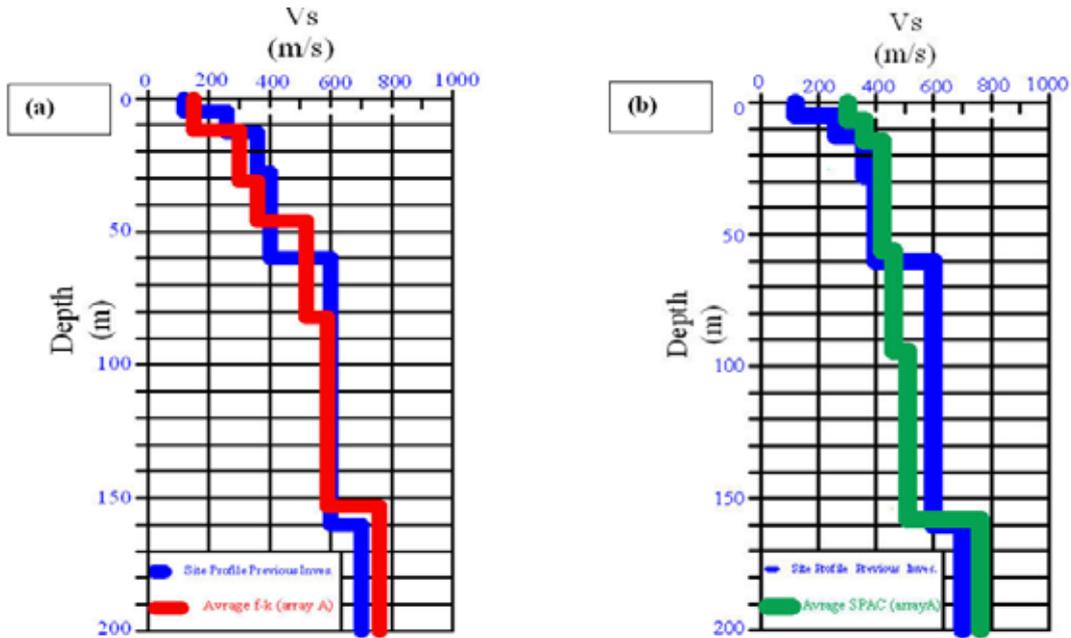


Fig. 7 Final results of Vs average profile compared with site profile, (a) using array A and f-k methods, (b) array A using SPAC method

Based on obtained average results from f-k processing method, it seemed that Vs profile derived from array A is more consistent with previous site investigation results (Fig. 7). The results of E-array

(using f-k method) in several available times compared to the reference Vs profile (In- Hole investigations) are presented in Figure 8-a; Also for array A (SPAC) in Figure 8-b.

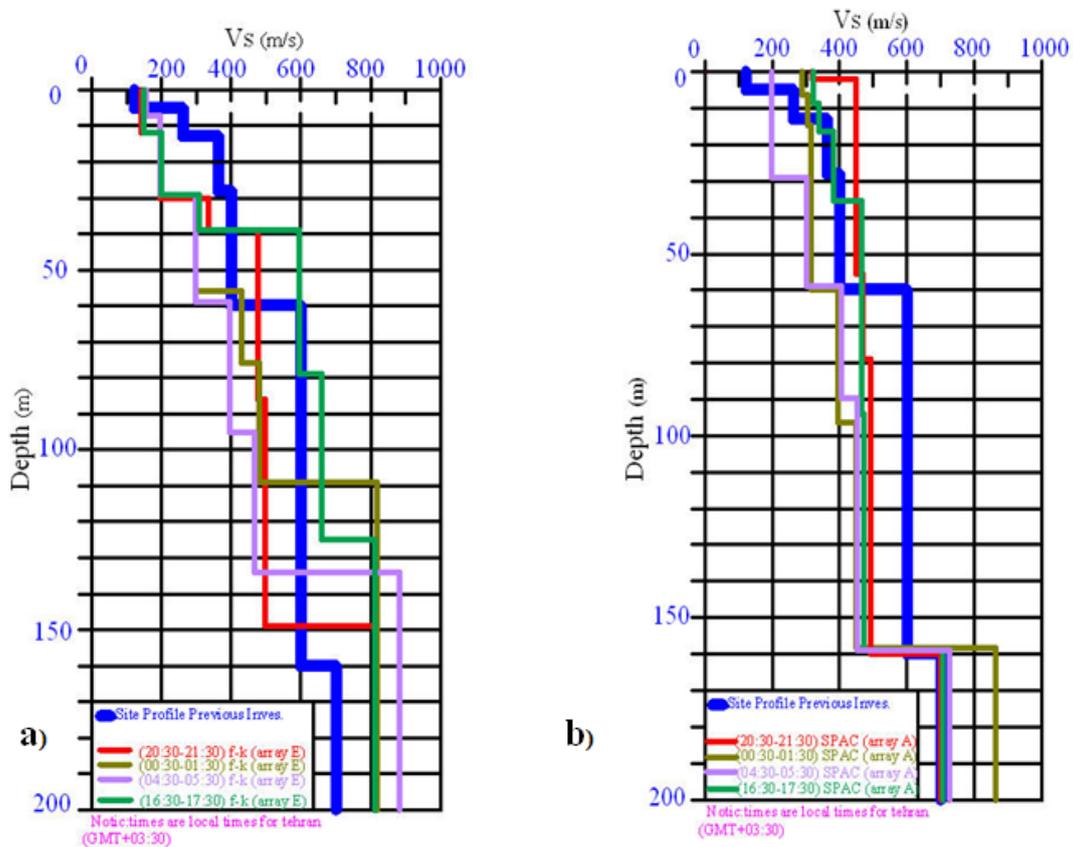


Fig. 8 Comparison of the Vs profile obtained using the microtremor recorded in different hours. The thick blue line show the previous investigations result a) array E (f-k method); b) array A (SPAC method)

**4. Evaluating the precision values**

With regards to this fact that there are 4 different arrays which processed using both SPAC and f-k methods in several time windows, data comparison were difficult. Depth of each layer and shear wave velocity of them are to main factors which extracted from Vs profiles. With averaging to comparison of data these two factors will be missed cause in averaging some details will be removed. In this paper an innovative method used for comparing Vs profiles obtained from SPAC and f-k methods with direct methods results. In this method two error defined; first, boundary detect error (BDE) that quantify the difference between obtained profiles with reference profiles in determination depth of layers and second, velocity detect error (VDE) which quantify the deviation of each profile in estimating shear wave velocity of layers respect to reference profile. The mathematical definition of these two errors is presented as follow:

- (Boundary detect Error ≡ Eb) :

$$E_b = \left( \sum_{i=m}^n |\Delta Z_i| \right) : (Z_{n+1} - Z_{m+1}) \times 100 \quad (4.1)$$

$$\Delta Z_i = Z_i - Z'_i \quad Z' = \sum H'$$

Where, follow to Figure 9:

$H_i$ : The thickness of each layer (Site profile, pervious investigations);

$H'_i$ : The thickness of each layer (Microtremor data inversion);

- (Velocity detect Error ≡ Ev) :

$$E_v = \frac{\sum \left( \frac{V_{S_i} - V_{S'_i}}{V_{S_i}} \right) \cdot H_i}{\sum H_i} \quad (4.2)$$

Where, follow to Figure 9:

$V_{S_i}$ : The Shear wave velocity of each layer (Site profile, pervious inv.);

$V_{S'_i}$ : The Vs of each layer (Microtremor data inversion).

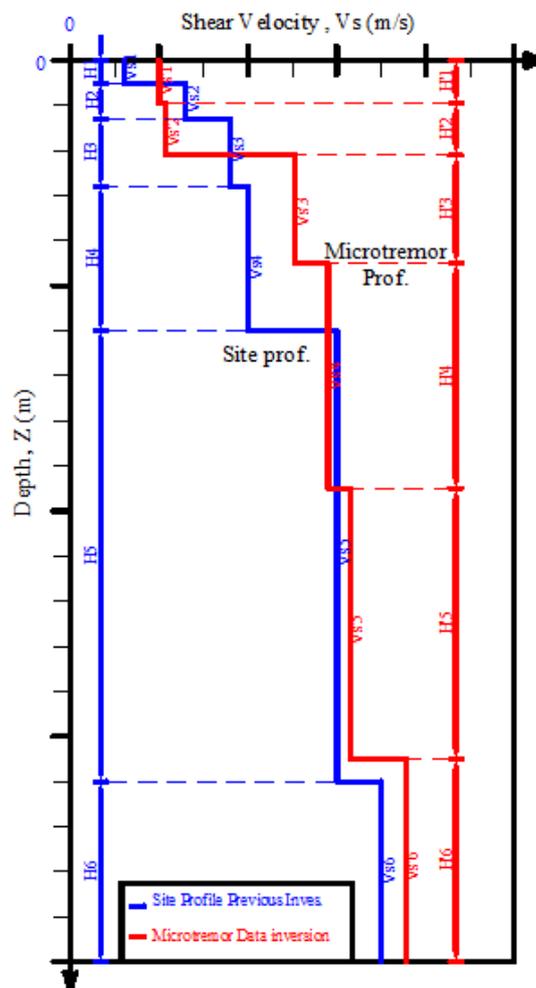


Fig. 9 Defining of parameters (Depth, Vs ...) to calculating Eb , Ev. ↑

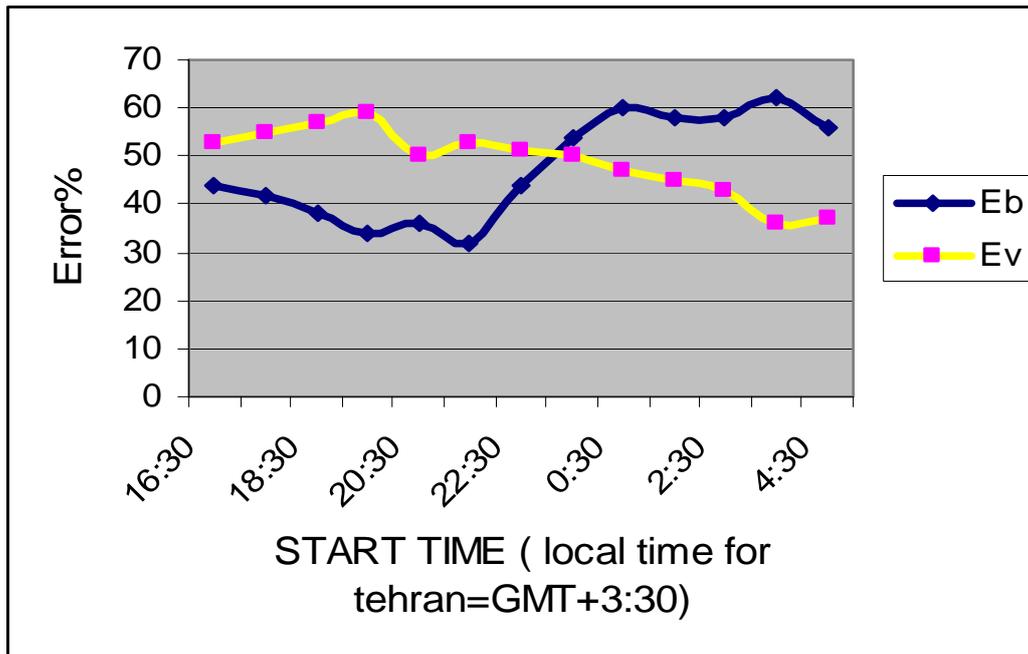


Fig. 10 The variation of two Errors/ Precisions in different recording times for 0-200m depth, array E

## 5. Discussion and coussion

In this paper capability of microtremor array method in defining shear wave velocity of subsurface soil has been investigated at a site in south-west of Tehran. To this end, first, using geotechnical drilling and direct geophysical method include down-hole and PS logging tests, layering and Vs profile of the studied site determined with high accuracy. Second, with installing some seismometers in different array layout including circular and triangular shape, microtremor data were recorded in the site. Afterward, microtremor data were processed using conventional f-k and SPAC methods and the result compared with direct methods.

The following results have obtained, with respect that all arrays in this study are in small array category, with radius's 25m, 35m and 50m.

1. Reliability of f-k method almost in the all arrays is more than SPAC method as it can be inferred from Figures 7 and 8.

2. Sharp layer changes have been detected in the near of surface ground (lower than 5 meters equal whit 20% smallest array radius(array A))

3. According to Figure 8 which showed Vs profiles in different time windows it could be found that accuracy of detecting the depth of layer in the low traffic times was more than high traffic times in the both f-k and SPAC methods. It might due to effect of these noises on local wave field.

4. Referring back to Figure 7 this result could be drawn that estimated Vs in the time windows that

ambient and human noises with high frequencies were prevailed, is more reliability (especially in the ranges of 1 to 2.5 times radius of array).

5. In the f-k method, estimated Vs values up to depth equal 2.5 times array radius were nearly 20% lower than reference Vs profile of the site (from PS-Logging), but in deeper depth the estimated Vs values are larger than reference Vs values. This result is not valid for SPAC method.

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